Homework 2  
Transaction Systems, SS2016  
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Exercise 1

\[ s = r_1(x) \ r_3(x) \ w_3(y) \ w_2(x) \ c_3 \ r_4(y) \ w_4(x) \ c_2 \ r_5(x) \ c_4 \ w_5(z) \ w_1(z) \ c_5 \]

Now calculate the Herbrand semantics without substitution of copier:

\[
\begin{align*}
H_s[z] &= H_s[w_1(z)] = f_{1z}(H_s[r_1(x)]) = f_{1z}(H_s[w_0(x)]) = f_{1z}(f_{0x}()) \\
H_s[y] &= H_s[w_3(y)] = f_{3y}(H_s[r_3(x)]) = f_{3y}(H_s[w_0(x)]) = f_{3y}(f_{0x}()) \\
H_s[x] &= H_s[w_4(x)] = f_{4x}(H_s[r_4(y)]) = f_{4x}(H_s[w_3(y)]) = f_{4x}(f_{3y}(f_{0x}()) = \text{see above}
\end{align*}
\]

Transactions \( t_3 \) and \( t_4 \) have copier as a function: \( f_{3y}(\alpha) = \alpha \) and \( f_{4x}(\alpha) = \alpha \). Thus deduce the final semantics

\[
\begin{align*}
H_s[z] &= f_{1z}(f_{0x}()) \\
H_s[y] &= f_{0x}() \\
H_s[x] &= f_{0x}()
\end{align*}
\]

Exercise 2

\[ s = r_1(x) \ r_3(x) \ w_3(y) \ w_2(x) \ r_4(y) \ w_4(x) \ c_2 \ r_5(x) \ c_3 \ w_5(z) \ c_5 \ w_1(z) \ c_1 \]

CSR — NO

Schedule is not conflict-serializable, since conflict graph of \( s \) is cyclic. See table 1.
Table 1: Some conflicts of schedule

<table>
<thead>
<tr>
<th>Edge</th>
<th>Conflict</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1, t_2$</td>
<td>$r_1(x)$ and $w_2(x)$</td>
</tr>
<tr>
<td>$t_2, t_4$</td>
<td>$w_2(x)$ and $w_4(x)$</td>
</tr>
<tr>
<td>$t_4, t_5$</td>
<td>$w_4(x)$ and $r_5(x)$</td>
</tr>
<tr>
<td>$t_5, t_1$</td>
<td>$w_5(z)$ and $w_1(z)$</td>
</tr>
</tbody>
</table>

FSR — YES

First, write operations of each transactions separately in table 2. Then calculate the Herbrand semantics of $s$ (the same as in the previous exercise) and derive rules for ordering (see table 3). Gather the rules: $t_3 < t_2 < t_4$ and $t_5 < t_1 < t_2$. Then there are only three possible serial schedules:

- $t_5 \ t_1 \ t_3 \ t_2 \ t_4$
- $t_5 \ t_3 \ t_1 \ t_2 \ t_4$
- $t_3 \ t_5 \ t_1 \ t_2 \ t_4$

It can be easily verified, that the Herbrand semantics of them are the same as $H[s]$.

Table 2: Transactions content

<table>
<thead>
<tr>
<th>TA</th>
<th>Content</th>
<th>Writes to</th>
<th>Reads from</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1$</td>
<td>$r_1(x) \ w_1(z)$</td>
<td>$c_1$</td>
<td>$z$</td>
</tr>
<tr>
<td>$t_2$</td>
<td>$w_2(x)$</td>
<td>$c_2$</td>
<td>$x$</td>
</tr>
<tr>
<td>$t_3$</td>
<td>$r_3(x) \ w_3(y)$</td>
<td>$c_3$</td>
<td>$y$</td>
</tr>
<tr>
<td>$t_4$</td>
<td>$r_4(y) \ w_4(x)$</td>
<td>$c_4$</td>
<td>$x$</td>
</tr>
<tr>
<td>$t_5$</td>
<td>$r_5(x) \ w_5(z)$</td>
<td>$c_5$</td>
<td>$z$</td>
</tr>
</tbody>
</table>

Table 3: Serial order deduction

<table>
<thead>
<tr>
<th>Data</th>
<th>$H_s[Data]$</th>
<th>Order rules</th>
</tr>
</thead>
</table>
| $x$ | $f_{4x}(f_{3y}(f_{0x}()))$ | $t_2 < t_4$ since $w_4(x)$ should be final for $x$
| | | $t_3 < t_4$ since $t_4$ uses $y$ from $t_3$
| | | $t_3 < t_2$, $t_4$ since $t_3$ uses $x$ from $t_0$
| $y$ | $f_{3y}(f_{0x}())$ | $t_3 < t_2$, $t_4$ since $t_3$ uses $x$ from $t_0$
| $z$ | $f_{1z}(f_{0x}())$ | $t_5 < t_1$ since $w_1(z)$ should be final for $z$
| | | $t_1 < t_2$, $t_4$ since $t_1$ uses $x$ from $t_0$